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Light bending in radiation background

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Abstract. We consider the velocity shift of light in presence of radiation emitted by a black body. Within geometric optics formalism we calculate the bending angle of a light ray when there is a gradient in the energy density. We model the bending for two simplified cases. The bending angle is proportional to the inverse square power of the impact parameter ($\propto 1/b^2$) when the dilution of energy density is spherically symmetric. The bending angle is inversely proportional to the impact parameter ($\propto 1/b$) when the energy density dilutes cylindrically. Assuming that a neutron star is an isothermal black body, we estimate the order of magnitude for such bending angle and compare it with the bending angle by magnetic field.

Keywords: neutron stars, gravitational lensing

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Contents

1 Introduction

Photons traveling in quantum electromagnetics (QED) vacuum in presence of electrically neutral medium such as background electromagnetic field, thermal environment, etc, have modified lightcone condition, i.e., $v \neq c$. The velocity shift can be described as the index of refraction in geometric optics.

In classical optics a light ray can be bent if there is a gradient in the refractive index. Previously we have computed the bending angles of light when the refractive index is nonuniform by external electric and magnetic fields [1, 2]. Study on the thermal radiation of neutron star in connection with its high magnetic field is an interesting issue for the demonstration of vacuum birefringence [3, 4]. In this work we consider the light bending when the vacuum is non-trivial by thermal radiation. Our purpose in this paper is to model the simplest case of the light bending induced by thermal radiation of compact astronomical objects.

2 Velocity shift and index of refraction

Classically the interaction of a light ray with electric or magnetic field is prohibited by the linearity of classical electrodynamics. However in QED it is possible by the vacuum polarization that allows the photon to exist as a virtual e^+e^--pair via which the external field can couple. The first study on nonlinear effect in the presence of an external electromagnetic field was performed by Euler and Heisenberg [5]. The low energy effective lagrangian describing the physics of such nonlinear interaction is described by [5, 6], in SI units,

$$\begin{aligned}\mathcal{L} &= -\frac{c^2\epsilon_0}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2\hbar^3\epsilon_0^2}{90m^4c}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right] \\ &= \frac{\epsilon_0}{2}(\mathbf{E}^2 - c^2\mathbf{B}^2) + \frac{2\alpha^2\hbar^3\epsilon_0^2}{45m^4c^5}\left[(\mathbf{E}^2 - c^2\mathbf{B}^2)^2 + 7c^2(\mathbf{E}\cdot\mathbf{B})^2\right]. \end{aligned} \tag{2.1}$$

The first order correction to the speed of light in the presence of external elctromagnetic field from the above lagrangian is given by [7–10]

$$\frac{v}{c} = 1 - \frac{a\alpha^2\hbar^3\epsilon_0}{45m^4c^3}\left[\mathbf{u}\times\mathbf{E}/c + \mathbf{u}\times(\mathbf{u}\times\mathbf{B})\right]^2, \tag{2.2}$$

where $\mathbf{u}$ denotes the unit vector in the direction of photon propagation, and a is a constant that depends on the photon polarization. Dittrich and Gies [11–13] developed a general formula to find the velocity shifts and refractive indices for soft photons traveling in QED vacuum modified by external media based on an effective action approach. The light cone condition under homogeneous media can be described by

$$k^2 = Q < T^{\mu\nu} > k_\mu k_\nu, \tag{2.3}$$

where $< T^{\mu\nu} >$ is the expectation value of the energy-momentum tensor in the modified vacuum and Q is the so-called effective action charge that depends on the parameters of the effective action.

One can represent the light cone condition in terms of velocity of light by choosing a certain reference frame and introducing

$$\bar{k}^\mu = \left(\frac{k^0}{|\mathbf{k}|}, \hat{\mathbf{k}}\right) = (v, \hat{\mathbf{k}}), \tag{2.4}$$

where $v = k^0/|\mathbf{k}|$ is the phase velocity. Then the equation (2.3) can be written as

$$v^2 = 1 - Q < T^{\mu\nu} > \bar{k}_\mu \bar{k}_\nu. \tag{2.5}$$

When the correction is small, $Q < T^{00} > \ll 1$, the speed of light averaged over the propagation direction is given by

$$v^2 = 1 - \frac{4}{3} Q < T^{00} > = 1 - \frac{4}{3} Q\rho, \tag{2.6}$$

where ρ is the energy density of general non-trivial QED vacuum.

The above formalism can be applied to all-loop effective actions. In the weak field limit, the two-loop corrected velocity shift averaged over polarization and propagation direction is obtained as [11]

$$v = 1 - \frac{4\alpha^2}{135 m^4}\left(11 + \frac{1955}{36}\frac{\alpha}{\pi}\right)\left[\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)\right]. \tag{2.7}$$

To the leading order (one-loop), this agrees with the result calculated from Euler-Heisenberg lagrangian.

Let us calculate the velocity shift when a light ray passes the non-trivial vacuum induced by the energy density of electromagnetic radiation. The energy momentum tensor of incoherent radiation (real photons) can be described by

$$T^{\mu\nu} = \rho U^\mu U^\nu, \tag{2.8}$$

where ρ is the energy density of photons emitted by radiation background and U^μ is a null propagation vector in the direction of radiation. If the radiation is spherically symmetric, $U^\mu = (1, 1, 0, 0)$ in spherical polar coordinate system. The light cone condition (2.3) can be written as

$$k^2 = Q\rho(U \cdot k)^2 = Q\rho(k_0 - k_r)^2. \tag{2.9}$$

The effective action charge Q can be calculated from the box diagram interaction [7]

$$Q_\pm = -\frac{a_\pm \alpha^2 \hbar^3}{45 m^4 c^5}, \tag{2.10}$$

where $a_+ = 14$ and $a_- = 8$ depending on the polarization. The velocity shift and the index refraction at one-loop level is given by

$$\frac{v}{c} = \frac{1}{n} = 1 - \frac{a_\pm \alpha^2 \hbar^3}{90 m^4 c^5} \rho \left(1 - \frac{k_r}{|\vec{k}|}\right)^2. \tag{2.11}$$

Note that there is the birefringence effect since the index of refraction depends on the polarization. The polarization of the photon is not fixed throughout the trajectory. Even if the incident photon started in one of the modes (pure perpendicular or pure parallel), the outgoing photon polarization is neither in pure perpendicular nor in pure parallel mode. Since the splitting occurs continually on every branch-out rays, the splitting results in the initial single light ray branching out to a light bundle. The bending angles calculated with $a_\pm$ give an envelope for the maximal and minimal bending angles. Since we are interested in the order-of-magnitude calculation and comparison with the bending angle by magnetic field, we use the index of refraction averaged over the polarization with $a_{\rm av} = 11$,

$$n = 1 + \frac{11 \alpha^2 \hbar^3}{90 m^4 c^5} \rho \left(1 - \frac{k_r}{|\vec{k}|}\right)^2. \tag{2.12}$$

3 The trajectory equation

In geometric optics, one can calculate the bending of light ray if the index of refraction is varying continually. The trajectory of light ray for a continually varying index of refraction can be written as [1, 2]

$$\frac{d\mathbf{u}}{ds} = \frac{1}{n}(\mathbf{u} \times \nabla n) \times \mathbf{u}, \tag{3.1}$$

where s denotes the distance parameter of the light trajectory with $ds = |d\vec{\mathbf{r}}|$ and $\mathbf{u} = d\vec{\mathbf{r}}/ds$ is the unit vector of the light ray. When the correction to the index of refraction due to the non-trivial vacuum is very small, the trajectory equation can be approximated to the leading order as

$$\frac{d\mathbf{u}}{ds} = (\mathbf{u}_0 \times \nabla n) \times \mathbf{u}_0, \tag{3.2}$$

where $\mathbf{u}_0$ denotes the initial direction of the incoming ray. For a ray coming in from $x = -\infty$ and moves to $+x$ direction,

$$\mathbf{u}_0 = (1, 0, 0), \tag{3.3}$$

and defining $\nabla n \equiv (\eta_1, \eta_2, \eta_3)$, the trajectory equations for $y(x)$ and $z(x)$ to the leading order are given by

$$\frac{d^2 y}{dx^2} = \eta_2, \qquad \frac{d^2 z}{dx^2} = \eta_3. \tag{3.4}$$

4 Bending by a spherical blackbody

Now we consider the trajectory of a light ray if there is a gradient in the energy density of radiation. As a source of the lens, we consider two simple cases when the gradient is spherically symmetric or cylindrically symmetric. Let us consider the spherically symmetric case first. Such case can be made by the dilution of energy density thermally radiated from the surface of a compact star. In general the temperature of any astronomical object may be

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different for different surface points. For example, the temperature of a magnetized neutron star on the magnetic pole is higher than that on the equator due to magnetic field effects. However, for simplicity, we will consider the mean effective surface temperature as a function of radius assuming that a neutron star is emitting energy isotropically as a black body in steady state.

The energy density of free photons emitted by a black body at temperature T is given by the Stefan's law

$$\rho = \frac{\pi^2}{15\hbar^3 c^3}(k_B T)^4, \tag{4.1}$$

where k_B is the Boltzmann's constant. The dilution of energy density as a function of radius can be obtained from the conservation of total radiation power, $L = 4\pi r_0^2 \rho_0 = 4\pi r^2 \rho$,

$$\rho(r) = \rho_0 \frac{r_0^2}{r^2}, \tag{4.2}$$

where r_0 is the radius of a spherical black body and ρ_0 the energy density of a spherical black body with surface temperature T_0 at r_0. The index of refraction, to the leading order, is given by

$$n(r) = 1 + \frac{11\pi^2\alpha^2}{1350}\left(\frac{k_B T_0}{mc^2}\right)^4 \frac{r_0^2}{r^2}\left(1 - \frac{k_r}{|\vec{k}|}\right)^2. \tag{4.3}$$

The electron mass energy mc^2 can be replaced by $k_B T_c$ with the critical temperature of QED defined as $T_c = mc^2/k_B = 5.94 \times 10^9$K.

Taking the direction of the incident ray as $+x$ axis on the xy plane ($r = \sqrt{x^2 + y^2}$), the index of refraction can be written as

$$n(r) = 1 + \frac{11\pi^2\alpha^2}{1350}\left(\frac{T_0}{T_c}\right)^4 \frac{r_0^2}{r^2}\left(1 - \frac{x}{r}\right)^2, \tag{4.4}$$

and the trajectory equation at the leading order is obtained as

$$y'' = \frac{22\pi^2\alpha^2}{1350}\left(\frac{T_0}{T_c}\right)^4 r_0^2 y \left(-\frac{3}{r^4} + \frac{3x}{r^5} + \frac{2y^2}{r^6}\right). \tag{4.5}$$

For the incoming photon with the impact parameter b, the initial condition reads

$$y(-\infty) = b, \qquad y'(-\infty) = 0. \tag{4.6}$$

Integrating eq. (4.5) with $y = b$ for the leading order solution, we obtain

$$y'(x) = -\frac{22\pi^2\alpha^2}{1350}\frac{T_0^4}{T_c^4}\frac{r_0^2}{b^2}\left[\frac{3}{4}\left(\tan^{-1}\frac{x}{b} + \frac{\pi}{2}\right) + \frac{b^3}{(b^2+x^2)^{\frac{3}{2}}} + \frac{3bx}{4(b^2+x^2)} - \frac{xb^3}{2(b^2+x^2)^2}\right], \tag{4.7}$$

$$y(x) = b\left[1 - \frac{22\pi^2\alpha^2}{1350}\frac{T_0^4}{T_c^4}\frac{r_0^2}{b^2}\left\{\frac{3}{4}\frac{x}{b}\left(\tan^{-1}\frac{x}{b} + \frac{\pi}{2}\right) + \frac{b^2}{4(b^2+x^2)} + \frac{x}{\sqrt{b^2+x^2}} + \frac{7}{4}\right\}\right]. \tag{4.8}$$

The total bending angle $\varphi_{\rm sph}$ can be obtained from $y'(\infty) = \tan\varphi_{\rm sph} \simeq \varphi_{\rm sph}$,

$$\varphi_{\rm sph} = -\frac{11\pi^3\alpha^2}{900}\frac{T_0^4}{T_c^4}\frac{r_0^2}{b^2}. \tag{4.9}$$

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5 Bending by a cylindrical blackbody

Now we consider the cylindrically symmetric case. Taking the axis of cylinder as z-axis, from the conservation of the radiation power, the dilution of energy density is given by

$$\rho(r) = \rho_0 \frac{r_0}{r}, \tag{5.1}$$

where $r = \sqrt{x^2 + y^2}$ and r_0 is the radius of the cylindrical black body. The refractive index can be written as

$$n(r) = 1 + \frac{11\pi^2\alpha^2}{1350}\left(\frac{T_0}{T_c}\right)^4 \frac{r_0}{r}\left(1 - \frac{x}{r}\right)^2, \tag{5.2}$$

For a ray moving to the $+x$ direction in the xy plane, the leading order trajectory equation can be written as

$$y'' = \frac{11\pi^2\alpha^2}{1350}\left(\frac{T_0}{T_c}\right)^4 r_0 y\left(-\frac{4}{r^3} + \frac{4x}{r^4} + \frac{3y^2}{r^5}\right). \tag{5.3}$$

For the impact parameter b, the solution is obtained as

$$y'(x) = -\frac{11\pi^2\alpha^2}{1350}\frac{T_0^4}{T_c^4}\frac{r_0}{b}\left[\frac{4}{3}\left(\frac{x}{\sqrt{b^2+x^2}} + 1\right) - \frac{1}{3}\frac{b^2 x}{(b^2+x^2)^{\frac{3}{2}}} + \frac{2b^2}{b^2+x^2}\right], \tag{5.4}$$

$$y(x) = b\left[1 - \frac{11\pi^2\alpha^2}{1350}\frac{T_0^4}{T_c^4}\frac{r_0}{b}\left\{2\left(\tan^{-1}\frac{x}{b} + \frac{\pi}{2}\right) + \frac{4}{3}\frac{\sqrt{b^2+x^2}+x}{b} + \frac{1}{3}\frac{b}{\sqrt{b^2+x^2}}\right\}\right]. \tag{5.5}$$

The total bending angle obtained from $y'(\infty)$ is

$$\varphi_{\rm cyl} = -\frac{44\pi^2}{2025}\alpha^2\frac{T_0^4}{T_c^4}\frac{r_0}{b}. \tag{5.6}$$

6 Application

Note that the bending angles in (4.9) and (5.6) depend on the factor $(T_0/T_c)^4$. Since the maximal value of T_0 on ground laboratory and a normal star like sun is of the order $T_0 \sim 10^3$K while $T_c \sim 10^9$K, detecting the bending by radiation on ground experiment or in the neighborhood of a normal star seems very difficult. As an application to possible real physical phenomena, let us consider the light bending by a magnetized neutron star since the surface temperature of neutron stars are pretty high and there is no atmosphere to prevent the propagation of light ray.

For a magnetized neutron star the light bending can also occur by both the magnetic field and gravitation. The bending by gravitational field is well-known from general relativity

$$\varphi_{\rm grav} = \frac{4G\mathcal{M}}{bc^2}. \tag{6.1}$$

The bending by magnetic field can be calculated with the index of refraction obtained by Euler-Heisenberg lagrangian [14, 15]. The bending by a magnetic dipole generally depends on the orientation of the magnetic dipole relative to the direction of the incoming ray and we have computed a general formula on the bending angles before [2]. We consider the case when the ray is passing the axis of dipole where the bending is maximal

$$\varphi_{\rm mag} = \frac{41\pi}{3\cdot 2^7}\frac{a\alpha^2\epsilon_0 c\hbar}{e^2}\frac{B_0^2}{B_c^2}\frac{r_0^6}{b^6}, \tag{6.2}$$

where $a = 8$ or 14 depending on the polarization of the photon, B_0 is the magnetic field at the surface of neutron star, and B_c is the critical magnetic field of QED defined by $B_c = m^2c^2/e\hbar = 4.4 \times 10^9$T.

Note that, from $\varphi_{\rm mag} \propto 1/b^6$ and $\varphi_{\rm sph} \propto 1/b^2$, the bending by magnetic field is dominant at short distance while the bending by the dilution of real photon density is dominant at long distance. The power dependence on the impact parameter, surface temperature, and magnetic field is imprinted from the energy density through the index of refraction $\nabla n \propto \nabla u$. For the bending by a magnetic dipole, $u \propto B_0^2/r^6$, and for the bending by a spherical black body, $u \propto T_0^4/r^2$.

To do an order-of-magnitude estimation, we consider the possible bending angles as functions of impact parameter. The mass of neutron star is of the order of solar mass so we take $\mathcal{M} \sim \mathcal{M}_{\rm sun} = 2 \times 10^{30}$kg. We take the radius of neutron star as $r_0 = 10$km. Most of the neutron stars possess surface magnetic field of the order $B_0 = 10^4 - 10^9$T. Of course there are neutron stars with the surface magnetic field above the QED critical limit known as the magnetars. However, we do not consider such extremely strong magnetic field since the calculation in (6.2) is based on the Euler-Heisenberg lagrangian. Thus we consider the surface magnetic field up to the order of 10^8T so that we take $B_0/B_c \sim 10^{-1}$ as the upper bound. The surface temperature of neutron stars is estimated as $T_0 \leq 10^6$K [16–18] and we take $T_0/T_c \sim 10^{-3}$ as the upper bound of the temperature. The bending angles for the above values are estimated as

$$\varphi_{\rm grav} \sim \mathcal{O}(10^{-1}) \times \frac{r_0}{b}; \qquad \varphi_{\rm mag} \sim \mathcal{O}(10^{-5}) \times \left(\frac{r_0}{b}\right)^6; \qquad \varphi_{\rm rad} \sim \mathcal{O}(10^{-17}) \left(\frac{r_0}{b}\right)^2. \tag{6.3}$$

The bending by magnetic dipole field dominates the bending by radiation for $b < 10^3 r_0$, while the bending by radiation dominates the magnetic bending for $b > 10^3 r_0$. However, both bending angles are still small compared with the gravitational bending.

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References

[1] J.Y. Kim and T. Lee, *Light bending in a Coulombic field*, *Mod. Phys. Lett.* **A 26** (2011) 1481 [arXiv:1012.1134] [INSPIRE].

[2] J.Y. Kim and T. Lee, *Light bending by nonlinear electrodynamics under strong electric and magnetic field*, *JCAP* **11** (2011) 017 [arXiv:1101.3433] [INSPIRE].

[3] J.S. Heyl, N.J. Shaviv and D. Lloyd, *The high-energy polarization-limiting radius of neutron star magnetospheres. 1. Slowly rotating neutron stars*, *Mon. Not. Roy. Astron. Soc.* **342** (2003) 134 [astro-ph/0302118] [INSPIRE].

[4] M. van Adelsberg and R. Perna, *Soft X-ray Polarization in Thermal Magnetar Emission*, *Mon. Not. Roy. Astron. Soc.* **399** (2009) 1523 [arXiv:0907.3499] [INSPIRE].

[5] W. Heisenberg and H. Euler, *Consequences of Dirac's theory of positrons*, *Z. Phys.* **98** (1936) 714 [physics/0605038] [INSPIRE].

[6] J.S. Schwinger, *On gauge invariance and vacuum polarization*, Phys. Rev. **82** (1951) 664 [INSPIRE].

[7] Z. Bialynicka-Birula and I. Bialynicki-Birula, *Nonlinear effects in Quantum Electrodynamics. Photon propagation and photon splitting in an external field*, Phys. Rev. **D 2** (1970) 2341 [INSPIRE].

[8] S.L. Adler, *Photon splitting and photon dispersion in a strong magnetic field*, Annals Phys. **67** (1971) 599 [INSPIRE].

[9] J.S. Heyl and L. Hernquist, *Birefringence and dichroism of the QED vacuum*, J. Phys. **A 30** (1997) 6485 [hep-ph/9705367] [INSPIRE].

[10] V.A. De Lorenci, R. Klippert, M. Novello and J.M. Salim, *Light propagation in nonlinear electrodynamics*, Phys. Lett. **B 482** (2000) 134 [gr-qc/0005049] [INSPIRE].

[11] W. Dittrich and H. Gies, *Light propagation in non-trivial QED vacua*, Phys. Rev. **D 58** (1998) 025004 [hep-ph/9804375] [INSPIRE].

[12] H. Gies and W. Dittrich, *Light propagation in non-trivial QED vacua*, Phys. Lett. **B 431** (1998) 420 [hep-ph/9804303] [INSPIRE].

[13] H. Gies, *Light cone condition for a thermalized QED vacuum*, Phys. Rev. **D 60** (1999) 105033 [hep-ph/9906303] [INSPIRE].

[14] V.I. Denisov, I.P. Denisova and S.I. Svertilov, *The Nonlinear electrodynamic bending of the X-ray and gamma-ray in the magnetic field of pulsars and magnetars*, Dokl. Akad. Nauk Ser. Fiz. **380** (2001) 435 [astro-ph/0110705] [INSPIRE].

[15] V.I. Denisov, I.P. Denisova and S.I. Svertilo, *Nonlinear electrodynamic effect of ray bending in the magnetic-dipole field*, Dokl. Phys. **46** (2001) 705.

[16] D. Page, *Surface temperature of a magnetized neutron star and interpretation of the ROSAT data. I. Dipolar fields*, Astrophys. J. **442** (1995) 273 [astro-ph/9407015] [INSPIRE].

[17] D. Page and A. Sarmiento, *Surface temperature of a magnetized neutron star and interpretation of the ROSAT data. II.*, Astrophys. J. **473** (1996) 1067 [astro-ph/9602042] [INSPIRE].

[18] A.Y. Potekhin, V. Urpin and G. Chabrier, *The magnetic structure of neutron stars and their surface-to-core temperature relation*, Astron. Astrophys. **443** (2005) 1025 [astro-ph/0508415] [INSPIRE].

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